G-estimation of structural nested models

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Conditional causal effect

- In this chapter introduce g-estimation method to estimate conditional causal effect.
- G-estimation is estimate conditional average causal effect through structural nested model.

Conditional causal effect

- A : Smoking cessation
- Y : Weight gain
- L : Confounder (Ex moderate exercise, male, white etc)
- Estimate the average causal effect of treatment A within levels of L.

$$E(Y^{a=1}|L) - E(Y^{a=0}|L)$$

$$E(Y^{a=1} - Y^{a=0}|L)$$

Conditional causal effect

- If there were no effect-measure modification by L, structural model for the conditional causal effect would be

$$E(Y^a - Y^{a=0}|L) = \beta_1 a$$

- Causal effect which is depend on L

$$E(Y^a - Y^{a=0}|L) = \beta_1 a + \beta_2 aL$$

Structural nested mean model

- Under conditional exchangeability $Y^a \perp A | L$, the structural model can also be written as
$$E(Y^a - Y^{a=0} | A = a, L) = \beta_1 a + \beta_2 aL$$
- Structural nested mean models are semiparametric because there is no parameter β_0 about intercept and β_3 for a term $\beta_3 L$.
- Leaving there parameters unspecified, structural nested models make fewer assumptions and can be more robust to model misspecification.

Rank preservation

- Suppose there are two lists of individuals ordered from larger to smaller value of the corresponding counterfactual outcome.
- If both lists are in identical order we say that there is rank preservation.
- When the effect of treatment A on the outcome Y is exactly the same, on the additive scale, for all individuals in the study population, we say that additive rank preservation holds.

Rank preservation

- An example of an rank-preserving structural model is additive conditional rank-preserving model.

$$Y_i^a - Y_i^{a=0} = \psi_1 a + \psi_2 a L_i \text{ for all individuals } i$$

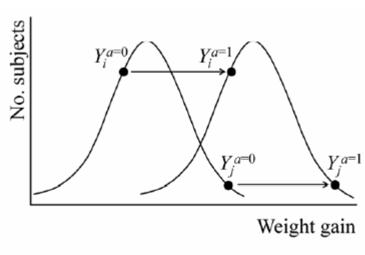

Figure 14.1

G-estimation

- Suppose there is effect modification by some components V of L.
- Assume that the conditional additive rank-preserving model

$$Y_i^a - Y_i^{a=0} = \psi_1 a + \psi_2 a V_i$$

- Write model in the equivalent form

$$Y^{a=0} = Y^a - \psi_1 a - \psi_2 a V$$

- By consistency

$$Y^{a=0} = Y - \psi_1 A - \psi_2 A V$$

- Define

$$H(\psi_1^*, \psi_2^*) = Y - \psi_1^* A - \psi_2^* A V$$

G-estimation

- If $\psi_1^* = \psi_1, \psi_2^* = \psi_2$, $H(\psi_1^*, \psi_2^*) = Y^{a=0}$
- Consider the logistic model

$$\text{logit} Pr(A = 1|Y^{a=0}, L) = \alpha_0 + \alpha_1 Y^{a=0} + \alpha_2 L$$

- Because of the conditional exchageability, $\alpha_1 = 0$.

G-estimation

- Consider the logistic model

  $\text{logit}Pr(A = 1|H(\psi_1^*, \psi_2^*), L) = \alpha_0 + \alpha_1 H(\psi_1^*, \psi_2^*) + \alpha_2 H(\psi_1^*, \psi_2^*)V + \alpha_3 L$

- Estimate ψ_1, ψ_2 as make both α_1 and α_2 0. (Grid search)

Non-rank preservation

- ψ_1 and ψ_2 produce a consistent estimate of the parameter β_1 and β_2 of the mean model if average treatment effect is equal in all levels of L.
- This is true regardless of whether the conditional additive rank preservation holds.
- G-estimation algorithm only requires that $H(\beta_1, \beta_2)$ and $Y^{a=0}$ have the same conditional mean given L.

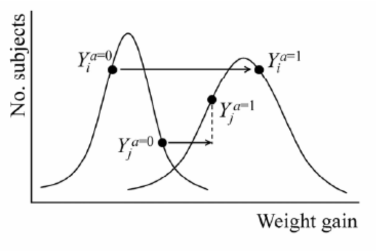

Figure 14.3

Selection Bias

- In the presence of censoring, causal effect is not $E(Y^{a=1} - Y^{a=0}|A, L)$ but $E(Y^{a=1,c=0} - Y^{a=0,c=0}|A, L)$
- G-estimation can only be used to adjust for confounding not selection bias.
- Thus when using g-estimation, one first needs to adjust for selection bias due to censoring by IP weighting.